The analysis of heuristics decision-making in abstract algebra proofing

by Junarti Junarti

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Abstract. Heuristics decision-making is the path, the shortcut of the human mind in making an immediate decision on the process of problem-solving in abstract algebra proofing. Heuristics decision-making are classified into four components, i.e. representative, availability, adjustment and anchoring. This study combined a quantitative and qualitative approach. A test was done twice in order to measure the five stages of verification, and questionnaires were also used to measure heuristics decision and were analyzed descriptively. The results of this study were (1) the students having representative heuristics were able to write the verification well; those having availability heuristics were able to prove when they were given the similar example with the problem; those having adjustment heuristics were able to prove with the ideas and concepts that have been understood; and those having anchoring heuristics cannot control the mistakes made in symbols writing, key ideas, steps, conditions of proof, and reasons supporting theorem; and (2) from the interview, it was revealed that the proof can be done if there are similar examples and with the help of friends, the students did not have a fully learning independence, the students were less confidence in making heuristics decision.

1. Introduction

The proof is an important topic in the field of mathematics curriculum and important aspects of mathematics competence [1-5]. For students, the problem of proof or verification is considered difficult [6] to be resolved. Students are accustomed to the inductive problems experienced in high school, so when they become college students,they are still influenced by the previous customs done in high schools. There is a misunderstanding among undergraduate students in writing mathematical proof. Stavrou [7] shows that students collectively make four recurring errors, i.e. the assumption of conclusions to prove conclusions, proving general statements using specific examples, do not prove both conditions in biconditional statements, and misusing definitions. The difficulties in establishing mathematical verification tasks [8] and writing mathematical proof [2] were also encountered by students.

The mathematical proof is an important topic in the teaching of mathematics, even though students have great difficulty in producing proof [1]. A study that helps the way of learning and understanding of students in proofing an abstract algebra done by Findell [9] can be used as an alternative solution. In addition, there are various stages of verification process which have been introduced by some experts, such as Polya [10] who proposes four stages of verification process, Bransford [11] proposes five stages of verification processthat are abbreviated in the word "IDEAL", Selden &Selden [5]

propose four aspects of writing a proof, and Fukawa-Connelly [13] summarize the aspects of proof-writing from Selden & Selden [5] in which there are five aspects of writing proof.

Other studies which indicated that each teacher has and uses a different way of verification of each case [13, 14] can be used as a way to help students in understanding and verifying an abstract algebra. In addition to the various ways introduced in the previous studies in problem-solving, Fukawa-Connelly [13] developed a sense of group and individual responsibility to contribute to the development of mathematical knowledge that has been known [15]. The knowledge of students' ability to write proof in abstract algebra is limited [2]. Abstract algebra has a different structure of verification analysis [9].

Decision-making, in this case, is defined as a tendency to overcome the problems experienced when there is more than one way for certain objects that are considered to meet the requirements [17] related to the concept required on problem-solving. The concept of decision-making is also defined by [15] as a decision, and any form of judgment that affects action; make selection among alternatives to get people to the desired results; determine the sanctions be applied to address a problem; judgment process as a result of assessment; the process of judging through the interpretation or comparison of information related to an event or a problem.

Heuristics decision-making in this study is the path or shortcut of our mind [15]. For example, if you and the people around you always have fast food, then your level for making decisions about fast food is high [15]. This heuristics decision merely refers to thinking [15]. Heuristics decisions refer to the thinking of objects that are similar to each other or have similar characteristics [18]. Thus, the heuristics in this study is the path of the human mind to make quick decisions, and has a margin of error and also correctness at the same time [18] in problem solving, especially in the process of verifying the abstract algebra of students in Mathematics Education Department.

This study explains the problem-solving ability and heuristics decision making of students in Mathematics Education Departmentin verifying the abstract algebra with five aspects of Fukawa-Connelly [13] proof writing which is studied through four types of heuristics decisions.

This study explains the problem-solving ability on the abstract algebra's proving with 5-aspect of Fukawa-Connelly's proof-writing. It reviews through 4 kinds of heuristics decisions.

2. Methods

This study used the combination of qualitative and quantitative approach. The quantitative design was using ANOVA test, while the qualitative design was using a Case Study Design [16]. In this study, the researcher used a case study aiming at the intensive study of decision making at the five stages of an aspect of writing proof on abstract algebra.

The data collection technique in this study was done through: 1) written test, to provide answers to solve the problem of proof with 5 aspects of the writing proof which was given to the subjects twice in two weeks; 2) questionnaires, which were used to record heuristics decision-making data conducted in solving the problem of proof; 3) Interview, which was conducted to three students. They were randomly interviewed from the second stage of the job description of the students with the following criteria, such as who does not write much in five aspects taken; who wrote the first, second and third aspects; who wrote down all of the aspects.

Since there were two kinds of data, i.e. quantitative data and qualitative data, the data analysis were also diffferent. The quantitative data were analyzed by using ANOVA, while the qualitative data were analyzed descriptively.

The role of the researcher in qualitative research serves as a key instrument in the interview data and fills the questionnaire of heuristics decision-making and in analyzing the measurement of the five aspects of the writing of the proof.

Abstract algebra was chosen as the mathematical content of this research because it is an important material in the curriculum of Undergraduate Program in Mathematics Education Department. Thus, the respondents of this study were the student of the sixth semester of Mathematics Education Department of IKIP PGRI Bojonegoro who had already taken Abstract Algebra Course 1 and are

taking Abstract Algebra Course 2. The number of respondents who were tested was 30 from 42 registered students.

3. Results and Discussion

3.1 Results

The result of quantitative data analysis yielded from the test which was used to test the difference of four types of heuristics decision-making on problem-solving abilities with ANOVA test by using SPSS is value of sig = 0.000 is smaller than the level = 0.05, then H_0 is rejected which means H_1 is accepted. So, there is a difference in problem-solving of the four types of heuristics decision on abstract algebra proof.

Furthermore, the SPPS output results were analyzed to know which type of heuristics decisions is the best and the worst.The result is presented in table 1 below.

Table 1. Descriptive Statistics of the Four Types of Heuristics Decision-making

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Min	Max
					Lower Bound	Upper Bound		
Representative	30	47.7000	11.85298	2.16405	43.2740	52.1260	20.00	66.00
Availability	30	39.8333	15.15457	2.76683	34.1745	45.4921	11.00	63.00
Adjustment	30	40.4000	14.19713	2.59203	35.0987	45.7013	18.00	63.00
Anchoring	30	24.8333	8.63467	1.57647	21.6091	28.0576	11.00	60.00
Total	120	38.1917	15.07254	1.37593	35.4672	40.9161	11.00	66.00

From table 1 above, it can be seen that from the four types of heuristics decisions, the mean value of Representative is 47.70, followed by Adjustment, Availability, and Anchoring. Form this result, it can be inferred that the decision making that mostly the students do is representative which means that the students are able to show the process of analysing the choices when making decisions well, through evaluating the advantages and disadvantages of the choices, and make a very remarkable decisions, at the end, focusing on special qualities by ignoring statistical information related to elements of a category and for highlighting similarities over other types.

The lowest type of decision is Anchoring heuristics. It means that the students have fewer desires in accordance with any references and reluctant to look for other references so that the students are immediately satisfied with any decision that has been made. This reveals that the students are in doubt to make a decisionfor an assessment by looking for a reference or targeted-problem faced without considering any information.

In this study, the verification process used five aspects of proof proposed by Selden & Selden [5] which are summarized by Fukawa-Connelly [13]as follows.

1) The Hierarchical Structure
2) The Construction Path
3) The Proof Framework
4) The formal-rhetorical part of a proof
5) The Problem-Centred Part of the Proof

The results of the data distribution of the students' answers on five aspects of the writing of proof from four abstract algebra questions in the first test are presented shown in Table 2.

Table 2. The data distribution of five aspects of writing proof on abstract algebra from the first test

No.	The Five Aspects of Writing Proof	Questions			
		1	2	3	4
1.	The Hierarchical Structure	S_2 S_{13} S_{14} S_{23} S_{25} S_{26} S_{27} S_{29}	S_2 S_{13} S_{14} S_{23} S_{25} S_{26} S_{27} S_{28} S_{29}	S_{13} S_{25} S_{26} S_{27} S_{29}	S_{13} S_{14} S_{23} S_{25} S_{26} S_{29}
		27%	30%	17%	20%
2.	The Construction Path	S_1 S_2 S_{13} S_{14} S_{21} S_{23} S_{26} S_{28} S_{29}	S_1 S_2 S_4 S_6 S_{13} S_{14} S_{23} S_{25} S_{26} S_{28} S_{29}	S_{13} S_{14} S_{21} S_{25} S_{26} S_{28} S_{29}	S_{13} S_{14} S_{23} S_{14} S_{25} S_{26} S_{29}
		30%	37%	23%	23%
3.	The Proof Framework	S_1 S_2 S_{12} S_{13} S_{18} S_{19} S_{20} S_{21} S_{23} S_{25} S_{28} S_{27} S_{28} S_{29} S_{30}	S_1 S_2 S_4 S_5 S_6 S_9 S_{11} S_{12} S_{13} S_{14} S_{15} S_{21} S_{25} S_{28} S_{28} S_{29}	S_2 S_{11} S_{12} S_{13} S_{14} S_{17} S_{18} S_{19} S_{21} S_{23} S_{25} S_{26} S_{27} S_{28} S_{29}	S_{13} S_{14} S_{21} S_{25} S_{25} S_{26} S_{29}
		50%	53%	50%	23%
4.	The Formal–Rhetorical part of a proof	S_1 S_2 S_3 S_4 S_{11} S_{12} S_{13} S_{18} S_{19} S_{20} S_{23} S_{25} S_{26} S_{27} S_{28} S_{29} S_{30}	S_1 S_2 S_4 S_5 S_6 S_{11} S_{13} S_{14} S_{15} S_{23} S_{25} S_{26} S_{27} S_{28} S_{29}	S_1 S_2 S_4 S_{13} S_{14} S_{15} S_{21} S_{23} S_{25} S_{26} S_{27} S_{28} S_{29}	-
		57%	50%	43%	0%
5.	The Problem-Centred part of the proof	S_1 S_2 S_3 S_4 S_8 S_{11} S_{12} S_{13} S_{14} S_{15} S_{16} S_{18} S_{19} S_{20} S_{21} S_{23} S_{24} S_{25} S_{26} S_{27} S_{28} S_{29} S_{30}	S_1 S_2 S_3 S_4 S_5 S_6 S_7 S_8 S_{11} S_{13} S_{14} S_{15} S_{20} S_{21} S_{23} S_{24} S_{25} S_{26} S_{27} S_{28} S_{29} S_{30}	S_1 S_2 S_3 S_4 S_5 S_6 S_7 S_8 S_{11} S_{13} S_{14} S_{15} S_{16} S_{20} S_{21} S_{23} S_{25} S_{26} S_{27} S_{28} S_{29}	S_{26}
		76%	73%	70%	3%

From the table 2 above, it is revealed that from 30 students, two students obtained the lowest score, they were S_9 and S_{16}. From the four questions, it is seen that question 4 was the question in which most students are not able to answer well. It can be seen that it is only 3% of students who could answer question 4 well. The Question 1 was answered by 76% of the students which means that the students' answers were analyzed based on the five aspects of proof [5]. From the distribution of answers to question 1, 2, and 3 above,it shows that the students tend to answer directly to the fifth aspect rather than the first aspect. The studentshave less ability to write good proof in the first and second aspects. The students generally like to write the aspect of direct proof based on what will be proofed without considering their previous knowledge (schemata).

In the first test results, students tend to provide solutions to proof than to write the proof on the third, fourth and fifth aspects. So after the students were given the test in the second time, it was obtained the following results presented in table 3.

Table 3. The data distribution of five aspects of writing proof on abstract algebra from the second test

No.	The Five Aspects of Writing Proof	Questions 1	2	3	4
1.	The Hierarchical Structure	S_1 S_2 S_3 S_4 S_5 S_{11} S_{12} S_{13} S_{14} S_{15} S_{16} S_{18} S_{19} S_{20} S_{21} S_{23} S_{24} S_{25} S_{26} S_{27} S_{28} S_{29} S_{30}	S_1 S_2 S_3 S_4 S_6 S_{11} S_{12} S_{13} S_{14} S_{15} S_{16} S_{18} S_{19} S_{20} S_{21} S_{23} S_{24} S_{25} S_{26} S_{27} S_{28} S_{29} S_{30}	S_1 S_2 S_3 S_4 S_{10} S_{11} S_{12} S_{13} S_{14} S_{15} S_{16} S_{18} S_{19} S_{20} S_{21} S_{23} S_{24} S_{25} S_{26} S_{27} S_{28} S_{29} S_{30}	S_1 S_2 S_3 S_4 S_8 S_{11} S_{12} S_{13} S_{14} S_{15} S_{16} S_{18} S_{19} S_{20} S_{21} S_{23} S_{24} S_{25} S_{26} S_{27} S_{28} S_{29} S_{30}
		76%	76%	76%	76%
2.	The Construction Path	S_1 S_2 S_3 S_4 S_8 S_{11} S_{12} S_{13} S_{14} S_{15} S_{16} S_{18} S_{19} S_{20} S_{21} S_{23} S_{24} S_{25} S_{26} S_{27} S_{28} S_{29} S_{30}	S_1 S_2 S_3 S_4 S_8 S_{11} S_{12} S_{13} S_{14} S_{15} S_{16} S_{18} S_{19} S_{20} S_{21} S_{23} S_{24} S_{25} S_{26} S_{27} S_{28} S_{29} S_{30}	S_1 S_2 S_3 S_4 S_8 S_{11} S_{12} S_{13} S_{14} S_{15} S_{16} S_{18} S_{19} S_{20} S_{21} S_{23} S_{24} S_{25} S_{26} S_{27} S_{28} S_{29} S_{30}	S_1 S_2 S_3 S_4 S_8 S_{11} S_{12} S_{13} S_{14} S_{15} S_{16} S_{18} S_{19} S_{20} S_{21} S_{23} S_{24} S_{25} S_{26} S_{27} S_{28} S_{29} S_{30}
		76%	76%	76%	76%
3.	The Proof Framework	S_{13} S_{14} S_{21} S_{25} S_{26} S_{28} S_{29}	S_1 S_2 S_4 S_5 S_6 S_7 S_{11} S_{12} S_{13} S_{14} S_{15} S_{21} S_{25} S_{26} S_{28} S_{29}	S_2 S_{11} S_{12} S_{13} S_{14} S_{17} S_{18} S_{19} S_{21} S_{23} S_{25} S_{26} S_{27} S_{28} S_{29}	S_{13} S_{14} S_{21} S_{23} S_{25} S_{26} S_{29}
		23%	53%	50%	23%
4.	The Formal–Rhetorical part of a proof	S_{13} S_{14} S_{28} S_{29}	S_1 S_2 S_4 S_5 S_6 S_{11} S_{13} S_{14} S_{15} S_{23} S_{25} S_{26} S_{27} S_{28} S_{29}	S_1 S_2 S_4 S_{13} S_{14} S_{15} S_{21} S_{23} S_{25} S_{26} S_{27} S_{28} S_{29}	S_{13} S_{14} S_{28} S_{29}
		12%	50%	43%	12%
5.	The Problem-Centred part of the proof	S_{13} S_{29}	S_1 S_{29}	S_1 S_2	S_{26} S_{29}
		5%	5%	5%	5%

From the five aspects of proof on the second test, the results show thatthe profile of the students' problem-solving ability in abstract algebra verification is described in the following distribution: the students' capability to verify until step 2 was 76%, 23% of them were able to verify until the step 3, 12% of them were able to verify until the step 4, and only 5% of them were able to verify until the step 5. In this second test, the students had shown significant changes at each stage of writing proof. There is an improvement compared to the first test. Yet, in the third, the fourth, and fifth stage there is a decline.

For the second test, there is a different distribution compared to the first one. This means that the focus of the students cannot spread evenly in every aspect of the writing of the proof. However, if each student is analysed, it can be seen that S_1 S_2 S_{13} S_{26} and S_{29}have a consistent answer by writing the aspect of proof both on the questions in the first stage and in the final stage. And those five students are always able to write the first, second, third, fourth, and fifth aspects of writing proof.

Furthermore, the result of the interview is as follows.

Student 1: The results of interviews on the first criterion showed that the student lacked the focus that spread in the five aspects because the student mainly focused on the questions. The student is confused and does not understand what is being asked of each aspect. Dealing with the heuristics decision made

by the student, it is clearly seen that the student was only able to write without thinking about the question. He was not able to connect between the instruction on the aspect of writing the proof and the problem to be proven. The student does not understand what is known and which will be proven. The student does not understand direct and indirect proof. The student often relies on answers from other friends rather than seeing examples. This type of student is classified as a student who does not have high learning independence.

Student 2: The student understands both the instruction from each aspect and the instruction of the questions, but the student was not able to link each component. The student understands the concept that will be proved and the known concepts as well as the supporting concepts or related concepts but was not able to describe the fourth and fifth aspects. In fact, if the results of the first test were checked, the student was able to write up to the fourth and the fifth aspect.This was because the student mainly focused on what will be proved without considering the situation of the first aspect. The student does not have confidence in the answer; he still depends on examples of answers that are similar and still depends on the answers of other friends.

Student 3: The answer categories of student who was interviewed show that the student was able to think from the various instructions in each question and from each aspect as well as a good understanding and mastery of concepts on abstract algebra. Thus, the student of this group has an understanding both in analytical and synthetical level well. Dealing with the heuristics decision-making questionnaire, this student has a good ability in representation skill and a good self-confidence by deciding the problem alone without helping from others. The student demonstrated their independence in making decisions on the selected proof from the various basic experiences.

3.2 Discussion

Fukawa-Connelly [13] asserts that writing proof in mathematics is important. And writing the proof requireshigh-order thinking skills according to the students' age [20] in which it is in the stage of formal thinking. Various factors influence the high-order thinking, one of them is that things which could inhibit learning the 'proof' are the belief that mathematical proof is a sequence of a step forward, deductive, systematic and logical that is formulated in formal mathematical language [1]. Another factor is the students do not understand the problems faced; this also affects why heuristics decisions in the proof are not done well in the form of representative, availability, adjustment, and anchoring. A comprehension is an essential requirement for students to determine the first step in the five aspects of what proofs are known and which ones will be proved.

In a proof, a good example helps students to make a proof correctly. The examples that have easy-to-understand solution steps and the final solution which is written in detail accompanied by the reasons may help the students to start experimenting with the same problem. Ideally, a solution of mathematical proofing process that does not reflect the real solution process will lead the student to perform only the final product of the verification process. Thus, using conventional examples will reinforce dysfunctional student's beliefs that proofis a direct deductive activity [1]. By using available examples will make a form of availability that students can use to take heuristics decisions on proving solutions. In addition, the examples available in the source book or given by the lecturers in writing different proof will also help the students [14].

3 Conclusions

The students having representative heuristics were able to write the verification well; those having availability heuristics were able to prove when they were given the similar example with the problem; those having adjustment heuristics were able to prove with the ideas and concepts that have been understood; and those having anchoring heuristics cannot control the mistakes made in symbols writing, key ideas, steps, conditions of proof, and reasons supporting theorem. From the interview, it was revealed that the proof can be done if there are similar examples and with the help of friends, the students did not have a fully learning independence, the students were less confidence in making heuristics decision.

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